Distributed Optimization and Learning: A Paradigm Shift for Power Systems

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Abstract

This survey provides a comprehensive overview of recent advances in distributed optimization and machine learning for power systems, particularly focusing on optimal power flow (OPF) problems. We cover distributed algorithms for convex relaxations and nonconvex optimization, highlighting key algorithmic ingredients and practical considerations for their implementation. Furthermore, we explore the emerging field of distributed machine learning, including deep learning and (multi-agent) reinforcement learning, and their applications in areas such as OPF and voltage control. We investigate the synergy between optimization and learning, particularly in the context of learning-assisted distributed optimization, and provide the first comprehensive survey of distributed real-time OPF, addressing time-varying conditions and constraint handling. Throughout the survey, we emphasize practical considerations such as data efficiency, scalability, and safety, aiming to guide researchers and practitioners in developing and deploying effective solutions for a more efficient and resilient power grid.

1. Introduction

The high penetration of distributed energy resources (DERs) introduces increased complexity and uncertainty into power system operation, raising concerns about power quality, voltage issues, stability, privacy, and cybersecurity (1–4). Distributed optimization and learning techniques offer a promising solution by enabling localized decision-making and parallel computation, potentially enhancing efficiency, data privacy, real-time response, and resilience to cyberattacks and component failures, while also alleviating communication bottlenecks. Machine learning (ML), including deep learning (DL) and reinforcement learning (RL), has emerged as a powerful tool for handling nonlinearities and uncertainties inherent in energy grids (5). The synergy between distributed optimization and ML holds the potential to revolutionize power system operations.

DERs: Small-scale power generation and storage units located near consumers (e.g., solar panels, wind turbines, batteries)

OPF: A fundamental optimization problem in power system that minimizes generation cost while meeting operational constraints

This survey aims to be a valuable resource for researchers and practitioners across power systems, control theory, optimization, and machine learning, offering insights into the application of distributed optimization and ML techniques to power system problems, particularly OPF. Building upon the foundation laid by existing comprehensive surveys on distributed optimization (e.g., (1–3, 6)) and ML (e.g.,(5)), we delve into the intersection of these fields with a focus on the following key aspects:

- A unified agent-based decomposition framework (Sec. 2) as a pedagogical tool to aid newcomers in understanding initial formulations of various distributed optimization algorithms for different power flow models (Sec. 3).
- Recent progress in addressing nonconvex problems (Sec. 3), focusing on methods with practical benefits for power systems, such as ADMM variants (with a focus on low per-iteration complexity, convergence acceleration, and handling non-ideal communication), ALADIN, and other promising approaches (e.g., distributed interior point and game-theoretic methods).
- ML Applications to distributed OPF, including distributed DL/RL (Secs. 4.1 and 4.2), multi-agent RL (MARL) (Sec. 4.3), and the synergy between optimization and ML in learning-assisted distributed optimization (Sec. 4.4).
- The first comprehensive survey of distributed real-time OPF (RT-OPF) (Sec. 5), highlighting connections to decomposition techniques (Sec. 3), the role of real-time measurements in algorithm design, and challenges/advances in handling time-varying conditions and constraints. We also explore potential cross-pollination with low per-iteration cost algorithms such as ADMM.

Throughout this survey, we emphasize the practical challenges of applying distributed optimization and ML techniques to power systems. We highlight how recent research addresses these challenges in various contexts, such as ensuring data efficiency and safety in deep learning for OPF (Sec. 4.1), synthesizing unified themes from the MARL literature in tackling non-stationarity, partial observability, and communication efficiency (Sec. 4.3), and guaranteeing safety and stability constraints in distributed RT-OPF (Sec. 5). By distilling key ideas from recent research advances, we aim to guide both researchers and practitioners. We conclude by identifying key challenges and future research directions (Sec. 6), including scalability, resilience, privacy, safety, robustness, and cybersecurity, to inspire further innovation in the field.

While this survey offers a comprehensive overview of the key areas outlined above, it does not cover all aspects of distributed optimization and ML for power systems. We refer readers to existing surveys for technical details on the original OPF formulation and its convex relaxations (e.g., (1, Sec. II-B), (7, Sec. II)), specific problem formulations for distribution systems (e.g., volt/var control and retail markets) (3), distributed optimization for discrete decision variables (6), and centralized RT-OPF methods (8). This focused scope allows us to provide an in-depth analysis of recent advancements in distributed methods while acknowledging the broader research landscape.

2. A Unified Distributed Formulation of OPF

2.1. Agent-based Decomposition and Consensus Formulation

Consider a power network represented as a graph $\mathcal{G}(\mathcal{N}, \mathcal{L})$. Let $\mathcal{N}$ be partitioned into K subregions $\mathcal{R}_1, \ldots, \mathcal{R}_K$, each managed by a local agent $k \in \mathcal{K} := \{1, \ldots, K\}$. Let x_k denote agent k's local variables, including its internal variables and a locally maintained estimate of shared/coupling variables with neighbors. This allows each agent to calculate the influence of other agents on itself (e.g., line flows). We use $x_k[i]$ to denote the i-th entry of x_k, $X[i,j]$ for the entry at the i-th row and j-th column of X, and $X[I]$ for the principal submatrix of X indexed by I. While each agent maintains its own estimate of the shared variables, these estimates should ensure consensus among neighboring agents:

$$\min_{\{x_k\}_{k \in \mathcal{K}}} \quad \sum_{k \in \mathcal{K}} f_k(x_k) \qquad \text{1a.}$$

$$\text{s.t.} \quad \sum_{k \in \mathcal{K}} A_k x_k = b, \qquad \text{1b.}$$

$$x_k \in \mathcal{X}_k \quad \forall k \in \mathcal{K}, \qquad \text{1c.}$$

where $f_k(x_k)$ is the local cost function. The local feasibility set $\mathcal{X}_k$ may include line flow and power balance equations, as well as limits on voltage magnitudes and generator outputs. The consensus constraint (Eq. 1b) ensures consistency among the shared variables. Without this constraint, or assuming the shared variables are fixed or measured by each agent, the problem decouples into K independent subproblems.

$\mathcal{G}(\mathcal{N}, \mathcal{L})$: Power network graph ($\mathcal{N}$: set of buses, $\mathcal{L}$: set of lines connecting the buses)

Consensus Constraint: $\sum_{k \in \mathcal{K}} A_k x_k = b$ enforces agreement on shared elements across agents, where A_k are derived from $\mathcal{G}$. E.g., to enforce $x_1[1] = x_2[1]$, set $A_1[1,1] = 1 = -A_2[1,1]$, and $b[1] = 0$ (all other entries in the first row of A_k are zero)

SDP Relaxation: Relax the rank-1 constraint on the voltage matrix $X = VV^*$ to a PSD constraint ($X \succeq 0$)

Rank-1 Solution: A solution X^* to the relaxed SDP problem such that $\text{rank}(X^*) = 1$, implying $X^* = VV^*$ for some voltage vector V (unique up to a global phase shift), thus solving the original AC OPF problem

2.1.1. Distributed Nonconvex Formulation. In <u>region-based decomposition</u> for AC OPF (e.g., (9)), the local variable x_k includes the real/reactive power generation, p_i and q_i, and complex voltage $v_i \in \mathbb{C}$ for all buses $i \in \mathcal{R}_k$; it also includes "local copies" of voltage v_j for all boundary buses $j \in \delta(\mathcal{R}_k)$, where $\delta(\mathcal{R}_k)$ is the set of buses outside of, but connected to, subregion $\mathcal{R}_k$ through tie-lines. The local objective f_k and constraint $\mathcal{X}_k$ can be nonconvex (9). In contrast, <u>component-based decomposition</u> (10, 11) considers every network component (generators, transformers, loads, transmission lines, etc.) as an agent.

2.1.2. Distributed SDP Relaxation. The semidefinite programming (SDP) relaxation of the OPF problem introduces a matrix variable $X \in \mathbb{H}^{|\mathcal{N}|}$ (where $\mathbb{H}$ is the set of $n \times n$ Hermitian matrices and $|\mathcal{N}|$ is the number of buses) to represent the outer product of the voltage phasors, i.e., $X = VV^*$, where $V \in \mathbb{C}^{|\mathcal{N}|}$ is the vector of complex numbers of size $|\mathcal{N}|$. The matrix X is symmetric and positive semidefinite (PSD) by construction. The SDP relaxation drops the rank-1 constraint on X, replacing it with the PSD constraint $X \succeq 0$. For the distributed formulation, each region can correspond to a "bag" of nodes in a tree/chordal/clique decomposition of $\mathcal{G}$ (12). For each agent $k \in \mathcal{K}$, the local variable x_k is the principal submatrix $X[\mathcal{R}_k] \in \mathbb{H}^{|\mathcal{R}_k|}$ of X, indexed by the buses in subregion

Chordal Sparsity: A property of power networks' graph structure that enables efficient decomposition of the OPF problem's SDP relaxation into smaller, coupled subproblems over a tree, facilitating scalable distributed computation

Synchronous/ Asynchronous Updates: Synchronous: agents update simultaneously using a global clock; easier to analyze. Asynchronous: agents update independently using local clocks; flexible and scalable for distributed systems

Event-Triggered Schemes: Updates occur only when triggered by certain events, reducing communication overhead

Time-Varying Communication Graphs: Communication links change over time, modeling dynamic networks and potential failures

$\mathcal{R}_k$. Within x_k, the consensus variables are the entries corresponding to the buses that appear in neighboring bags of the tree decomposition, i.e., separator sets. The consensus constraint (Eq. 1b) ensures consistency of these entries across bags. If the optimal solution to the SDP relaxation is rank-1, it solves the original OPF problem, i.e., the relaxation is exact. In practice, even with higher-rank solutions, near-optimal solutions to the OPF can often be constructed. The local constraint set $\mathcal{X}_k$, with the addition of the PSD constraint $X[\mathcal{R}_k] \succeq 0$, is convex under the SDP relaxation.

2.1.3. Distributed SOCP Relaxation. In the second-order cone programming (SOCP) relaxation, the power flow equations are reformulated using branch power flows and voltage magnitudes, which can be derived from the SDP relaxation by imposing additional constraints on the structure of X while removing the computationally demanding constraint $X \succeq 0$. SOCP constraints have a simpler structure and can be handled by specialized solvers. For radial (tree topology) networks, the SOCP relaxation is exact under mild conditions (13). For a radial network, each bus is an agent with local variable x_k including the squared voltage magnitude and the net complex power injection at bus k, as well as the branch power flow and the squared magnitude of the branch current from bus k to its ancestor; the local variable x_k also includes a local copy of the variables from its neighbors. The local constraint set $\mathcal{X}_k$ includes the SOCP constraints.

2.1.4. Communication Topology and Protocol. The communication topology, modeled as a directed/undirected graph, determines how agents exchange information. Common topologies include distributed networks with m-hop neighbors, where agents communicate only with neighbors within m hops, star networks with a central coordinator, and hierarchical networks with communication occurring between different levels. Information exchange protocol defines the type, frequency, and timing of shared data (e.g., primal/dual variables) and may involve synchronous/asynchronous updates, event-triggered schemes, and time-varying communication graphs (2, Sec. 4).

3. Non-Convex Distributed Optimization Techniques

3.1. ADMM and Variants

The distributed OPF formulation (Eq. 1) can be interpreted as a multi-block extension of the classical two-block ADMM, extending it to handle K blocks of variables $\{x_k\}_{k\in\mathcal{K}}$ with separable objectives $\{f_k\}_{k\in\mathcal{K}}$ and consensus constraint (Eq. 1b). While ADMM is naturally suited to address both local and consensus constraints, its direct multi-block extension may not be convergent (14), necessitating further modifications.

Problem 1 is suitable for primal decomposition, since fixing coupling variables decouples the problem into subproblems. Thereby, it can be reformulated as a two-block problem:

$$\min_{x,z} \sum_{k\in\mathcal{A}} f_k(x_k) \qquad \text{s.t.} \quad A_k x_k - z_k = b_k,\ x_k \in \mathcal{X}_k,\ \forall k \in \mathcal{K}, \quad \sum_{k\in\mathcal{K}} z_k = 0, \tag{2}$$

where we introduce the auxiliary variables $z = \{z_k\}_{k\in\mathcal{K}}$ to facilitate consensus update. This allows exploiting classical ADMM on the blocks x and z, with the primal update of x naturally decomposing across agents $k \in \mathcal{K}$. Also, the slack z update admits a closed-form solution, i.e., $z_k^{t+1} = A_k x_k^{t+1} - b_k - d^{t+1}$, for all $k \in \mathcal{K}$, where $d^{t+1} = \frac{1}{K}\left(\sum_{k\in\mathcal{K}} A_k x_k^{t+1} - b\right)$

is the average violation of the coupling constraint (Eq. 1b). The primal update is:

$$x_k^{t+1} = \arg\min_{x_k \in \mathcal{X}_k} \quad f_k(x_k) + (\lambda^t)^\top (A_k x_k) + \frac{\rho}{2}\|A_k x_k - A_k x_k^t + d^t\|^2, \quad \forall k \in \mathcal{K}, \qquad 3.$$

which can be performed in parallel by agents, and the dual update is: $\lambda^{t+1} = \lambda^t + \rho d^{t+1}$, where λ_k are the dual variables associated with (Eq. 1b) and $\rho > 0$ is a penalty parameter.

Problem 1 is also amenable to dual decomposition, because relaxing the coupling constraint decouples the problem into subproblems. We introduce Lagrange multipliers λ and formulate the Lagrangian function: $L(x, \lambda) = \sum_{k\in\mathcal{A}} f_k(x_k) - \lambda^\top(\sum_{k\in\mathcal{A}} A_k x_k - b)$ and the resulting dual problem: $\min_\lambda \sum_{k\in\mathcal{A}} f_k^*(A_k^\top \lambda) - b^\top \lambda$, where $f_k^*(z) = \sup_{x_k}\{z^\top x_k - f_k(x_k) : x_k \in \mathcal{X}_k\}$ is the Fenchel conjugate of f_k under the assumption of a bounded convex subset $\mathcal{X}_k$. This problem is well known as consensus optimization, where the sum of K objectives is coupled through the consensus variable λ, and can be handled by the classical two-block ADMM via introducing local copies of $\lambda = \{\lambda_k\}_{k\in\mathcal{K}}$ and linking variables $\zeta = \{\zeta_{ij}\}_{(i,j)\in\mathcal{L}}$:

$$\min_{\lambda,\zeta} \quad \sum_{k\in\mathcal{A}} \left(f_k^*(A_k^\top \lambda_k) - \frac{1}{K} b^\top \lambda_k \right) \quad \text{s.t.} \quad \lambda_i = \zeta_{ij}, \lambda_j = \zeta_{ij}, \ \forall (i,j) \in \mathcal{L}. \qquad 4.$$

Applying ADMM yields the dual consensus ADMM implementation (15), with the primal updates: $x_k^{t+1} = \arg\min_{x_k\in\mathcal{X}_k} \quad f_k(x_k) + \frac{1}{4\rho|\mathcal{N}_k|}\|A_k x_k - \frac{1}{K}b - w_k^t + \rho\sum_{j\in\mathcal{N}_k}(\lambda_j^t + \lambda_k^t)\|^2$ and $\lambda_k^{t+1} = \frac{1}{2\rho|\mathcal{N}_k|}(A_k x_k^t - \frac{1}{K}b - w_k^t) + \frac{1}{2|\mathcal{N}_k|}\sum_{j\in\mathcal{N}_k}(\lambda_j^t + \lambda_k^t)$, and the dual update $w_k^{t+1} = w_k^t + \rho\sum_{j\in\mathcal{N}_k}(\lambda_k^t - \lambda_j^t)$. This algorithm supports parallel computation among the agents and networked communication schemes.

The above variants of multi-block ADMMs, known as parallel ADMM (16, Ch. 3) and dual consensus ADMM (15), have been applied to cost allocation in peer-to-peer electricity markets (17), electrical vehicle (EV) coordination (18), black-start and parallel restoration (19), and distributed OPF (20). Other multi-block modifications include: 1) Proximal regularization: Adds terms of the form $\frac{1}{2}\|x_k - x_k^t\|_{P_k}^2$ to subproblems, with $P_k \succeq 0$. For example, proximal Jacobian ADMM (21) requires $P_k \succ \rho(K-1)A_k^\top A_k$ to be sufficiently large for convergence. Since the proximal coefficient matrices P_k are generally required to be linearly growing with the number of agents K, a slower convergence is likely for larger problems. 2) Prediction-correction ADMM: Variants such as Jacobian ADMM with correction step (22) generate a prediction using Jacobian ADMM and then use correction steps for convergence with $\mathcal{O}(1/t)$ iteration complexity. 3) Block-wise ADMM: Variants such as block-wise ADMM (23) artificially split variables into two groups and apply two-block ADMM, often with smaller proximal coefficients than proximal Jacobian ADMM, potentially leading to faster convergence. This is demonstrated in coordinated control for microgrid clusters (24).

Jacobian Decomposition: Enables parallel updates of agent variables (contrasting with sequential updates, a.k.a. Gauss-Seidal, in classical ADMM), but is generally less stable and may require proximal methods to control the approximation error to the joint primal update

3.1.1. Accelerated ADMMs.
Accelerated ADMMs aim to improve the convergence rate of classical ADMM, implying fewer iterations and communications. One notable example is fast ADMM (25), which introduces an interpolation step based on the Nesterov acceleration technique and achieves an $O(1/t^2)$ convergence rate for strongly convex problems, improving upon the $O(1/t)$ rate of classical ADMM. However, convergence analysis for more general problems remains open. As noted in (26), existing techniques modify the primal and dual sequences in the iterative process, uniformly characterized by $\hat{\omega}^{t+1} = \mathrm{acc}(\omega^{t+1}, \omega^t, \omega^{t-1}, \ldots)$, where $\omega^t = \{x^t, \lambda^t\}$ is the stack of primal and dual variable updates generated by an ADMM

Optimization Algorithms as Dynamical Systems

Many optimization algorithms can be viewed as discretizations of continuous-time dynamical systems, offering insights into algorithm design, convergence analysis, and real-time control (Sec. 5). For example, gradient descent, $x_{t+1} = x_t - \alpha \nabla f(x_t)$, approximates gradient flow: $\dot{x}(t) = -\nabla f(x(t))$. If f is convex, the Lyapunov function $\mathcal{V}(x) = f(x) - f(x^*)$ (with $x^* \in \arg\min f(x)$) has $\dot{\mathcal{V}}(t) = \nabla f(x)^\top \dot{x}(t) = -\nabla f(x)^\top \nabla f(x) \le 0$, ensuring monotonic convergence. Reverse engineering an optimization algorithm involves choosing parameters and discretization schemes (e.g., forward Euler, Runge-Kutta) to balance accuracy and complexity. Integral quadratic constraints (IQC) from robust control theory can be used to analyze and design iterative algorithms (32). For constrained problems ($\min f(x)$ s.t. $x \in \mathcal{X}$), projected gradient methods can be modeled by the differential inclusion: $\dot{x} \in -\nabla f(x) - N_{\mathcal{X}}(x)$, where $N_{\mathcal{X}}(x)$ is the normal cone to ensure feasibility. Similar dynamics exist for other constrained methods, including primal-dual methods with evolving primal and dual variables. Convergence analysis often uses generalized derivatives due to non-uniqueness of $\dot{\mathcal{V}}(t)$.

Per-Iteration Complexity: Computational effort per algorithm iteration. In ADMM, this is the effort spent by each agent solving its subproblem before communicating with others. Low per-iteration complexity often means one or few gradient steps, function evaluations, or closed-form solutions.

Non-Ideal Communication: Communication between agents or with a coordinator is subject to delays, packet loss, noise, or other imperfections.

or its variant at iteration t and "acc" represents an acceleration technique. A guard condition ensuring monotonic decrease of the combined residual is introduced for convergence (26). The second-order information of the dual function is exploited with a Newton step in (27) to accelerate the dual variable update. Connections have been drawn between accelerated ADMM variants and continuous-time dynamical systems, establishing convergence rates through Lyapunov analysis (28). Moreover, a second-order dynamical system with vanishing damping has been shown to yield various inertial parameter rules, including Nesterov acceleration, under suitable time discretization (29). Accelerated ADMMs have demonstrated effectiveness in applications such as distributionally robust economic dispatch (30) and AC/DC hybrid power networks (31).

3.1.2. ADMMs with Low Iteration Complexity. ADMM variants aim to reduce per-iteration complexity by using approximations and proximal terms (e.g., $\|x - x^t\|^2$ to control approximation accuracy). Linearized ADMM optimizes local linear approximations, leading to simpler updates such as projected gradient steps or proximal mappings (33, 34). For nonconvex problems, bounded primal and dual updates are typically required to construct a sufficiently decreasing and lower bounded Lyapunov function (34). Stochastic ADMM performs gradient-like iterates with noisy gradients of the augmented Lagrangian (AL) function (33), which is useful when explicit functions are unavailable; however, the high variances of stochastic gradients lead to a convergence rate gap: $\mathcal{O}(1/\sqrt{t})$ for stochastic ADMM versus $\mathcal{O}(1/t)$ for its deterministic counterpart. To address this issue, variance reduction techniques have been proposed, including a stochastic path-integrated differential estimator (35), further combined with acceleration techniques (36). For instance, (37) provides a unified framework for inexact stochastic ADMM covering several well-known algorithms. Convergence of these inexact variants usually requires the linearized objective components to be Lipschitz differentiable, as well as sufficiently large proximal coefficients to bound the errors caused by inexact updates.

3.1.3. ADMMs with Non-Ideal Communications. Modern power systems are susceptible to random link failures due to factors like network congestion, infrastructure failures, signal interference, cyber attacks, and intentional noise added for privacy. The study (20) found that ADMM performance in unbalanced distribution networks degrades significantly under high levels of communication failure and noise. To address this, several ADMM algorithms have been developed, incorporating flexible agent activation mechanisms or asynchronous updates. Asynchronous updates improve computational efficiency by reducing idle time caused by delays or packet losses (38). In these schemes, a master node sets a maximum tolerable delay τ for each worker, enforced by a delay counter. The master proceeds with updates upon receiving new information from a sufficient number of workers, while ensuring remaining workers do not exceed the delay bound. There is often a trade-off between the number of iterations and waiting time, influenced by the delay bound τ and partial synchronization mechanism (38). Communication problems can be modeled as a time-varying network with asynchronous updates (39). An asynchronous dual decomposition algorithm has been proposed and compared favorably with existing methods in coordinating DERs under communication asynchrony and computation errors (40). Additionally, a data server with its own clock cycles to handle asynchronous data exchange for local consensus has been used in (41) to replace the central aggregator in (38), potentially facilitating easier integration into communication networks.

3.1.4. Other Considerations.

A proximal ADMM variant has been developed that allows each agent to select its step size autonomously based solely on local information, independent of the communication topology (42). A scaled dual descent approach within the AL framework has been proposed to handle more general nonlinear equality constraints, offering improved theoretical complexity guarantees compared to previous methods (43).

3.2. Augmented Lagrangian Alternating Direction Inexact Newton (ALADIN)

ALADIN (44) addresses nonconvex problem (Eq. 1) by solving decoupled problems in primal variables, similar to ADMM, while also requiring an approximation of the constraint Jacobian and Hessian to solve a coupled Quadratic Programming (QP) problem.

Consider local constraints $\mathcal{X}_k = \{x_k : h_k(x_k) = 0, g_k(x_k) \le 0\}$ in (Eq. 1), where h_k and g_k are assumed to be twice continuously differentiable. The key steps involve solving decoupled problems to either local or global optimality for each agent $k \in \mathcal{K}$:

$$\min_{x_k} f_k(x_k) + \langle A_k x_k, \lambda \rangle + \frac{\rho}{2}\|x_k - x_k^t\|_{\Sigma_k}^2 \quad \text{s.t.} \quad g_k(x_k) \le 0, \quad h_k(x_k) = 0, \tag{5}$$

where $\rho \ge 0$ is a penalty parameter, $\Sigma_k \succ 0$ is a weighting matrix, and λ is the dual variable. After solving (Eq. 5), the approximations of the constraint Jacobian $\nabla \tilde{g}_k(x_k^{(t+1)})$ and Hessian $H_k \approx \nabla^2_{x_k x_k}(f_k(x_k^t) + \gamma_k^\top g_k(x_k^{t+1}) + \mu_k^\top h_k(x_k^{t+1}))$ are computed, where γ_k and μ_k are the Lagrange multipliers; compared to ADMM, the use of more accurate Hessian and Jacobian approximations can reduce iterations at the cost of increased per-iteration complexity. Subsequently, a coupled QP is solved at a central node:

$$\begin{aligned}\min_{\Delta x_k} \quad & \sum_{k\in\mathcal{K}} \frac{1}{2}\|\Delta x_k\|_{H_k}^2 + \langle \Delta x_k, \nabla f_k \rangle + \lambda\left(\sum_{k\in\mathcal{K}} A_k(x_k^t + \Delta x_k) - b\right) + \frac{\rho}{2}\left\|\sum_{k\in\mathcal{K}} A_k(x_k^t + \Delta x_k) - b\right\|_2^2 \\ \text{s.t.} \quad & \nabla\tilde{g}(x_k^t)\Delta x_k = 0.\end{aligned}$$

Finally, the primal and dual variables are updated as $x_k^{t+1} = x_k^t + \Delta x_k$ and $\lambda^{t+1} = \lambda^t + \rho\left(\sum_{k\in\mathcal{K}} A_k x_k^{t+1} - b\right)$. Under mild assumptions, ALADIN converges to a local minimizer of the nonconvex problem from any feasible starting point when combined with the proposed globalization strategy (44). Under suitable conditions, it achieves a quadratic or superlinear local convergence rate (44), matching centralized sequential QP methods.

ALADIN has been applied to AC OPF and power system analysis (45, 46), AC/DC hybrid systems (47, 48), and heterogeneous power systems in both single-machine numerical simulations (46) and geographically distributed environments (49). However, its increased per-step communication and scalability issues, particularly with inequality constraints, are drawbacks; (47) shows an improved ADMM outperforms ALADIN in scalability for the AC OPF problem. To address these, (45) employs approximation methods for H_k using blockwise and damped BFGS updates. Bi-level distributed ALADIN (50) eliminates the central coordinator in the coupled QP step by solving it with decentralized ADMM or conjugate gradient. Recent advancements include improved computing times for large-scale AC power flow problems using second-order corrections for linearization errors of active constraints in (Eq. 5) (51). Open-source software for distributed and decentralized ALADIN is also available (52).

3.3. Distributed Interior Point Method

The Distributed Interior Point Method (IPM) is a promising approach for solving the large-scale nonconvex OPF problem. To overcome the limitations of extensive communication and central coordination of existing distributed second-order methods (53), distributed IPM reformulates (Eq. 1) by replacing inequality constraints with logarithmic barrier terms in the objective function:

$$\begin{aligned} &\min_{\{x_k, s_k\}_{k\in\mathcal{K}}} \sum_{k\in\mathcal{K}} \left(f_k(x_k) - \mu \sum_{i=1}^{m_k} \ln(s_{k,i}) \right) \\ &\qquad \text{s.t.} \quad h_k(x_k) = 0, \quad g_k(x_k) + s_k = 0, \quad s_k \geq 0, \quad \forall k \in \mathcal{K}, \quad \text{and (Eq. 1b)} \end{aligned} \tag{6}$$

where $\mu > 0$ is the barrier parameter, $s_k = \{s_{k,j}\}_{j=1,\dots,m_k} \in \mathbb{R}^{m_k}$ are the slack variables,.

The main challenge in decentralization is solving the coupled linear system arising from the Karush-Kuhn-Tucker (KKT) conditions of (Eq. 6) in each Newton step. In (54), an incremental-oriented ADMM variant is presented for distributed OPF with discrete variables, consisting of outer-loop iterations based on an extended IPM and inner-loop iterations based on ADMM. The outer-loop extended IPM forms a regional linear correction equation with coupling relationships between neighboring areas, enabling the use of ADMM to compute primal-dual directions in a distributed manner. Another approach involves two-stage optimization, decomposing the power network into a master network and subnetworks (55). By smoothing subproblems with a barrier term, the second-stage value function becomes differentiable with respect to the master problem variables, allowing for efficient nonlinear solvers using primal-dual IPMs, which offer fast local convergence.

3.4. Game-Theoretic Methods

Potential games (PGs) are applicable for power systems due to their ability to model individual interests, handle nonconvex objectives and constraints, and guarantee convergence to Nash equilibrium (NE) under certain learning dynamics (57). In an OPF context, a potential function may incorporate generation costs and power balance penalties, e.g., $\phi(p^g_k, p^g_{-k}) = -\sum_{k\in\mathcal{K}} f_k(p^g_k) - \gamma \left| p^d + p^{\text{loss}} - \sum_{k\in\mathcal{K}} p^g_k \right|$, where γ is a penalty factor, p^g_k and

Game-Theoretic Perspectives on Distributed Optimization

Distributed NE seeking aims to find an NE where no agent can unilaterally improve its utility $u_k(x_k, x_{-k})$ (the negative of its cost function). Here, x_k are the agent's local variables (including estimates of shared variables), and x_{-k} are other agents' variables. In a potential game, with a properly designed potential function $\phi(x)$, the NE may correspond to the solution of the distributed optimization problem with the added consensus constraint to ensure consistent estimates of shared variables. Distributed NE seeking algorithms, such as leader-following consensus and passivity-based approaches, offer unique insights (56), including the strategic behavior of agents, the potential for incentive alignment, robustness against adversarial behavior, and accommodation of private and shared nonlinear constraints. Continuous-time NE seeking is particularly relevant for distributed RT-OPF (Sec. 5).

p^g_{-k} are the active power outputs for generator k and all others, respectively, p^d is the total demand, and p^{loss} is the total transmission loss. The utility function for each generator, $u_k\left(p^g_k, p^g_{-k}\right) = \phi\left(p^g_k, p^g_{-k}\right) - \phi\left(0, p^g_{-k}\right)$, aims to minimize individual cost while contributing to collective cost reduction and power balance. Game-theoretic methods have shown faster convergence and lower generation costs compared to some heuristics (57, 58). Notably, PGs offer a realistic representation of interactions among agents with potentially conflicting objectives, and handle non-cooperative behavior through mechanism design, aligning individual interests with the global objective. This is particularly relevant for integrating DERs (59). The local generalized NE concept and variational equilibrium proposed in (59) address nonconvexities from AC power flow constraints in a non-cooperative setting. Distributed NE seeking algorithms, especially under partial decision information, has close connection to distributed optimization algorithms (56).

Potential Game: A multiplayer game where there exists a function (i.e., potential function) ϕ such that: $u_k(x'_k, x_{-k}) - u_k(x''_k, x_{-k}) = \phi(x'_k, x_{-k}) - \phi(x''_k, x_{-k})$ for every $k \in \mathcal{K}$, $x'_k, x''_k \in X_k$, $x_{-k} \in \prod_{j \neq k} X_j$.

4. Distributed Machine Learning Techniques

4.1. Deep Learning for Distributed OPF and Related Problems

DL for distributed OPF often uses direct prediction approaches to approximate the mapping from grid conditions to control setpoints, known as solution functions (see, e.g., (60) for a connection to MPC and DL). Simulations demonstrate significant speedup with minor optimality loss and constraint violation (61). Supervised learning with penalty terms (61) or primal-dual methods (62) ensure feasibility. Decentralized approaches with local ML models, trained to predict the optimal setpoint based on local measurements, are developed (63). Incorporating uncertainty is vital due to renewable generation variability. This is addressed through chance constraints (64) or conditional value-at-risk (CVaR) (65), which provides a convex surrogate for chance constraints. Graph neural networks (GNNs) incorporate grid topology (66), with robustness against anomalous and missing measurements (67). Attention networks have been employed alone (68) or with Convolutional Neural Networks (CNNs) (69). Post-training, GNNs and CNNs enable distributed predictions using local computations based on limited neighboring information. Data-driven methods depend heavily on training data quality and coverage, with out-of-sample robustness and constraint satisfaction being key challenges due to frequent topology changes and DER uncertainty.

Solution Function: A mapping $\pi(z, \theta)$ that finds the optimal decision x minimizing the objective function $f_{(z,\theta)}(x)$ over feasible set $\mathcal{X}_{(z,\theta)}$, both parameterized by z and θ.

4.1.1. Data Efficiency and Scalability. Sobolev training enhances data efficiency by incorporating sensitivities of the OPF solution function into the regression process (70). Instead of the "OPF-then-learn" paradigm, decentralized policies can be directly integrated into the OPF problem ("OPF-and-learn"). For instance, (62) learns distributed nonlinear inverter controls using a deep neural network (DNN) with individualized inputs and partially connected layers, formulated under a chance-constrained framework and solved with gradient-free learning. To address scalability, (71) proposes a novel distributed approach that decomposes the power network into regions, first predicting coupling variables, then training region-specific models in parallel. This approach scales to large networks (up to 6700 buses) while maintaining feasibility and reducing training time.

4.1.2. Constraint Satisfaction. Feasibility can be ensured through various approaches: 1) Restricted feasible region during training: Modifying the OPF feasible region to encourage models to produce strictly feasible solutions (72). 2) Active set prediction: Predicting active constraints and solving a reduced DC OPF problem (73). 3) Physics-informed models: Incorporating physical constraints into the loss function via penalty terms (67–69). 4) Feasibility restoration: Post-processing or projection onto the feasible space using power flow solvers, e.g., predict-and-reconstruct in (61). 5) Implicit layers and gauge mappings: Embedding feasibility restoration within the model using projection (74) or gauge (one-to-one) mappings (75). 6) Control-theoretic safe synthesis: Applying control-theoretic tools to define a feasible set for neural network weights that satisfy constraints (76). Each approach has trade-offs. Methods 1 and 2 simplify learning but may struggle with complex feasible spaces or yield infeasible solutions. Methods 3 and 4 effectively reduce violations but lack strict guarantees. Techniques 5 and 6 offer principled feasibility embedding but can be computationally expensive or rely on assumptions. Most methods are agnostic to DL architecture and can be used to learn distributed policies with sparse connections (62).

4.1.3. Perspective from Algorithm Unrolling. Deep learning architectures, such as DNNs, CNNs, GNNs, or recurrent neural networks (RNNs), can be viewed as the repeated application of an operator $\mathcal{F}^{(l)}$ across multiple layers $l \in \{1, ..., n_l\}$, i.e., $\mathcal{F}^{(n_l)} \circ \mathcal{F}^{(n_l-1)} \circ \cdots \circ \mathcal{F}^{(1)}$. This is reminiscent of iterative algorithms. For a simple illustration, the iterative algorithm $x^{(l+1)} = \sigma_{\rho/\kappa}(x^{(l)} + \frac{1}{\kappa}B^\top(b - Bx^{(l)}))$ can be used to solve $\min \frac{1}{2}\|Bx-b\|_2^2 + \rho\|x\|_1$, where $\sigma_{\rho/\kappa}$ is the element-wise soft-thresholding function $\sigma_\theta(x) = \operatorname{sign}(x)\max(0, |x| - \theta)$ with $\theta = \rho/\kappa$ and κ usually taken as the largest eigenvalue of $B^\top B$. We can treat each iteration as an instantiation of the operator $\mathcal{F}_{ISTA}(x^{(l)}) = \sigma_{\theta^{(l)}}(W_1^{(l)}x^{(l-1)} + W_2^{(l)}b)$, where $(\theta^{(l)}, W_1^{(l)}, W_2^{(l)})$ are learnable parameters to train from data with $x^{(0)}$ as the initial point. This concept broadly connects to algorithm unrolling (77), where optimization algorithms are unrolled into DL architectures. By connecting to algorithm unrolling for algorithms such as ADMM (78), we can potentially develop distributed optimization algorithms that leverage the expressiveness and learning capabilities of DL models.

Algorithm Unrolling: Interpreting deep learning architectures as unrolled versions of iterative algorithms, where each layer corresponds to an iteration

Markov Decision Process (MDP): A tuple $(\mathcal{S}, \mathcal{A}, P, R, \gamma)$, with $\mathcal{S}$: state space, $\mathcal{A}$: action space, P: transition probability function, R: reward function, γ: discount factor

4.2. Distributed RL for OPF and Related Problems

Distributed RL is well-suited for power system optimizations such as OPF, which involve high-dimensional spaces, complex constraints, and real-time decisions. In distributed RL, K agents interact with the environment in parallel. At time step t, agent k observes state $s_t^k \in \mathcal{S}$, selects action $a_t^k \in \mathcal{A}$ according to policy π_{θ_k}, where θ_k represents the parameters of the policy, and receives reward $r_t^k = R(s_t^k, a_t^k)$. The environment transitions to state s_{t+1}^k according to $P(s_{t+1}^k|s_t^k, a_t^k)$. The goal is to find a set of policies $\{\pi_k\}_{k=1}^K$ that maximizes the expected cumulative discounted reward:

$$V(\{\pi_k\}_{k=1}^K) = \mathbb{E}_{\tau\sim p\left(\tau|\{\pi_k\}_{k=1}^K\right)}\left[\sum_{k=1}^K \sum_t \gamma^t r_t^k\right], \qquad 7.$$

where $\tau = \{s_t^k, a_t^k, r_t^k\}_{k,t}$ is the set of trajectory of states, actions, and rewards for all agents, and $p(\tau|\theta_1, \ldots, \theta_K)$ is the probability distribution over trajectories induced by $\{\pi_{\theta_k}\}_{k=1}^K$.

Various distributed RL algorithms, such as distributed Q-learning (79) and actor-critic methods (80), enable efficient policy updates through shared experiences. For instance, IMPALA (Importance Weighted Actor-Learner Architecture) (80) uses an actor-learner architecture with parallel data generation and centralized policy learning, reducing training time and enabling scalability to thousands of machines without sacrificing data efficiency. PQL (parallel Q-learning) (79) parallelizes key RL components, e.g., data collection, policy learning, and value learning, enhancing network update frequency. Recent work (81) suggests that using a single policy for parallel exploration can be effective, potentially simplifying the coordination of exploration policies. Recent studies (31, 82) demonstrate distributed RL's potential in complex OPF scenarios. (82) addresses sparse rewards and exploration through parallel exploration for Transient SC OPF. (82) addresses sparse rewards and exploration through parallel exploration for Transient Security-Constrained OPF. (31) introduces a scalable hierarchical RL framework for complex optimizations.

Applying distributed RL to OPF requires consideration of the parallel interaction assumption, as agents can influence each other's states and rewards. This setup can be relevant in simulation or weakly decoupled systems, but care is needed to avoid potential divergence due to distributional shifts (83). Despite these challenges, insights from distributed RL, such as parallelization, stable learning, and simplified exploration, can inform the development of realistic RL-based solutions for power systems.

Distributional Shift: A change in the data distribution encountered by a model, compared to training. In RL, this can arise from environment changes, policy updates, or multi-agent interactions, leading to degraded performance or instability. Connected to nonstationarity and out-of-distribution generalization.

4.3. Multi-Agent RL for Distributed OPF and Related Problems

MARL has shown promise for distributed optimization/control problems, where multiple agents coordinate actions in a shared environment (84). This can be formalized as Decentralized Partially Observable MDP (Dec-POMDP) (85).

In a fully cooperative Dec-POMDP, agents share a reward function and seek a joint policy $\pi = \{\pi_k\}_{k\in\mathcal{K}}$ that maximizes the expected discounted cumulative reward (Eq. 7) (84). In contrast, competitive or mixed Dec-POMDPs involve agents with individual reward functions R_k aiming to maximize their own expected discounted return $V_k(\pi_k, \pi_{-k}) = \mathbb{E}_{\tau\sim p\left(\tau|\{\pi_k\}_{k=1}^K\right)}\left[\sum_t \gamma^t r_k^t\right]$ while considering others' policies π_{-k}. The resulting joint policy $\pi^* = \{\pi_k^*\}_{k=1}^K$ represents a Nash equilibrium, where $\pi_k^* \in \arg\max_{\pi_k} V_k(\pi_k, \pi_{-k}^*)$.

Most works assume fully cooperative agents (84, 86–93), with a few considering coordination signal design (89, 94). Non-cooperative setting in power system applications has been examined in online feedback equilibrium seeking (56, 57, 95).

Dec-POMDP: Extends MDP to multi-agent settings with partial observability. Key additions: $\mathcal{A} = \times_k \mathcal{A}_k$: joint action space, $\{R_k\}_{k\in\mathcal{K}}$: agent-specific rewards, $\{\mathcal{O}_k\}_{k\in\mathcal{K}}$ individual observation spaces, O: observation probability function

Key challenges in MARL include nonstationarity, scalability, and partial observability (96, 97). Nonstationarity arises from concurrent policy updates, while scalability issues stem from the combinatorial growth of the joint action space. Partial observability necessitates efficient communication and coordination, as well as robustness. These challenges are particularly relevant for power systems, where agents (e.g., DERs, generators) with individual objectives, constraints, and limited information must coordinate actions. Nonstationarity and scalability can hinder convergence to an optimal solution, while partial observability may lead to suboptimal local decisions misaligned with the collective goal.

4.3.1. Dealing with Nonstationarity. Centralized Training for Decentralized Execution (CTDE) allows agents to share information during training but act based on local observations during execution (87–89, 93). MADDPG (Multi-Agent Deep Deterministic Policy Gradient) (98), a popular CTDE method, uses a centralized critic conditioned on all agents' observations and actions, while the actor only accesses local information. Although primarily applied to cooperative settings (e.g., (88)), MADDPG can also handle mixed cooperative-competitive environments. Off-policy learning enhances stability by learning from past experiences. Examples include MASAC (Multi-Agent Soft Actor-Critic) (90), off-policy maximum entropy RL (86), and Twin TD3 (Delayed Deep Deterministic Policy Gradient) (91, 99). Maintaining a model of other agents, as in MADDPG or via techniques like confederate image technology (93), is beneficial. (96) discusses five categories of handling nonstationarity, with common approaches in power systems being ignoring (assuming stationarity) and forgetting (updating based on recent observations), and a few works on responding/learning opponent models.

4.3.2. Scalability. To enhance scalability, parameter sharing is a common approach, where agents share network parameters for value function or policy estimation (88, 99). This allows leveraging data from all agents to update a single shared network, improving scalability and reducing policy oscillations. Combining parameter sharing with Graph Convolutional Networks (GCNs) can further incorporate topology information (88). Efficient exploration techniques, such as parameter space noise (92), can prevent premature convergence in large action spaces. Spatial discount factors (87) encourage agents to consider the impact of their actions on neighboring agents, limiting the state/action space span. Open-source simulation platforms, such as (84, 87), facilitate comparison of various MARL algorithms, with some, such as MADDPG and TD3, demonstrating good scalability (84).

4.3.3. Handling Partial Observability. Local measurements are commonly used to achieve distributed optimization under partial observability (84, 87, 88, 90). Recurrent networks, such as Gated Recurrent Units (GRUs) (e.g., (88)) and Long Short-Term Memory (LSTM) (e.g., (87)), can effectively encode history to extract relevant features. Learning a surrogate model using Sparse Variational Gaussian Processes (SVGP) to create a simulation environment for MARL (92) can reduce real-world communication and data collection. Agents modeling other agents (93, 98) can also mitigate partial observability.

4.3.4. Communication Efficiency. Communication allows agents to share information and coordinate actions, but it must be done efficiently. Some approaches assume no explicit communication, e.g., decentralized training (100). Selective communication is common, where agents only communicate a subset of relevant information, such as value/policy data (86) or encoded state information (87). In structured communication, such as networked MARL (e.g., (86, 91)), each agent only needs to communicate with its neighbors. Agents can also learn communication protocols end-to-end, such as using differentiable communication (87). To handle agent and communication failures, (86) proposes constructing replacement states using historical averages and the agent's own policy networks to maintain operations.

The impact of communication topology changes on learning performance is studied in (91).

Incentive mechanisms can be designed to encourage collaboration. A cooperative bi-level framework, introducing an asymmetric Markov game to align agent objectives and guide equilibrium behaviors, along with a bi-level actor-critic algorithm for real-time control, is proposed in (89). Similarly, (99) adopts a bi-level approach to balance operational safety and market participants' interests. While existing approaches use penalty functions and global reward signals to promote cooperation and align objectives, (94) introduces Markov Signaling Game, a framework for studying strategic incentive-compatible communication between a sender and a receiver. The signaling gradient and extended obedience constraints help learn efficient and stable policies under information asymmetry.

4.3.5. Other Considerations. (88) introduces a GCN into the multi-agent actor-critic framework to enable generalization to different grid topologies. For constraint satisfaction, (84) proposes a voltage barrier function, later adopted and extended by (88) and others.

Robustness ensures graceful performance degradation under disturbances or adversaries. PowerNet (87) has been evaluated for robustness against different levels of load variations and agent disconnection or addition during operation, showing that it maintains good performance and quickly adapts to topology changes. Imperfect observations and topology flexibility have been addressed by encoding topology status as continuous variables, enabling adaptation to reconfigurations (90). Robustness against anomalous measurements has been enhanced by combining spatial and temporal variation pattern extraction using attention mechanisms and trajectory history features (93).

4.4. Learning-Assisted Distributed Optimization Techniques

Integration of RL in Distributed Optimization Most optimization methods from Sec. 3 can be integrated with RL to tackle complex and stochastic nonlinear dynamic control problems. These include primal-dual decomposition and Lagrangian relaxation, where RL optimizes dual variables for faster convergence (101–104); interior-point policy optimization, integrating RL with IPM for effective constraint handling (105); and stochastic optimization, incorporating RL for managing uncertainties (103); and adaptive optimization (106), where RL is used to leverage and adapt the solution function of an optimization problem (60) as a policy function in a distributed setting.

Learning-Assisted ADMM for OPF Several studies have demonstrated the improved effectiveness of integrating ADMM with learning methods for solving OPF problems (107–109). An asynchronous ADMM framework with momentum-extrapolation prediction has been introduced to manage asynchronous updates and communication failures (107). RNNs have been applied to predict ADMM convergence rates, accelerating optimization while maintaining privacy (108). ADMM's consensus parameter learning can be learned, optimizing decentralized power system efficiency (109). Deep Q-learning has been employed to dynamically select optimal penalty parameters in ADMM for AC OPF, significantly reducing computational complexity (110).

5. Distributed Real-Time Optimal Power Flow and Related Problems

RT-OPF, also known as online OPF, addresses the challenges posed by increasing DER penetration by leveraging real-time grid data and continuously updating control settings (8). RT-OPF differs from standard OPF in several key aspects: 1) cost functions, constraints, and network parameters can be time-varying; 2) algorithms must track the optimal solution rapidly with implementable solutions (111); and 3) only a subset of variables are directly controllable, while others are determined implicitly by the power flow equations. Online optimization problems in power systems span various timescales, from sub-minute to minute timescale (frequency/voltage regulation), to minute to hour timescale (RT-OPF), to hours to day timescale (energy storage system scheduling and multi-stage economic dispatch) (8).

While earlier works laid the groundwork for RT-OPF (see (8) for a review), recent research has focused on distributed RT-OPF, which leverages local measurements for improved robustness against single point of failure and a plug-and-play architecture that eases the integration of new grid components (e.g., (112–117)). This approach has been demonstrated in a case study on a 502-node distribution system, where the calculation time was reduced to 2.34% of the centralized counterpart (117).

For distributed optimization, the transition from static to real-time involves incorporating real-time measurements of voltages, currents, and power flows at the point of common coupling, which essentially exploits the laws of physics to solve the power flow equation and information exchange. These measurements are used in primal-dual updates to calculate regulating signals (dual variables) for agent coordination and feasibility (112, 114, 118). They also play a role in correcting model inaccuracy suffered by open-loop feedforward control when computing the required gradients (sensitivities) of directly controllable variables to indirectly controllable variables. Typically, precomputed linearized power flow models are combined with real-time measurement feedback to effectively handle nonlinearities and avoid the centralized nonlinear power flow Jacobian in real-time (115, 117, 119).

5.1. Optimization Methods for Distributed RT-OPF

5.1.1. Distributed Formulation and Decomposition Methods. A key step from static to real-time optimization involves introducing time dependency in key parameters in (Eq. 1) and decomposing the problem for distributed optimization and optimal trajectory modeling.

As in Sec. 3, the main techniques include Lagrangian relaxation (LR) based decomposition, such as ADMM (112), dual ascent (114, 115, 119), and regularized Lagrangian of the convex relaxation (113). KKT-based decomposition, such as the distributed interior point method (116), and primal decomposition by duplicating coupling variables for each subsystem and imposing consensus constraints (120, 121) are also used.

Real-time measurements enable decoupling the sensitivities of power flow states among different areas, allowing each agent to predict its future power flow states using only local and aggregated information from neighbors (117). The changes in power flow states in each area k at time $t+1$ can be expressed as: $\Delta x_k(t+1) = S_{kk}(t) \times \Delta p_k(t) + \sum_{j\in\delta(k)} S_{kj}(t) \times \Delta p_j(t)$, where $\Delta x_k(t+1)$ represents the predicted changes in power flow states, $S_{kk}(t)$ is the sensitivity submatrix, $\Delta p_k(t)$ represents the changes in DER output powers, and $\sum_{j\in\delta(k)} S_{kj}(t) \times \Delta p_j(t)$ is the aggregated information from neighboring areas.

Another approach is to implicitly decompose the OPF problem by using learned local equilibrium functions $h_k^e(q_k, v_k)$, which map local reactive power q_k and voltage v_k to an approximation of the optimal reactive power setpoint q_k^* from the centralized OPF problem

(122). The enables a decentralized incremental control algorithm for each agent k is: $q_k(t+1) = q_k(t) + \epsilon(h_k^e(q_k(t), v_k(t)) - q_k(t))$, where $\epsilon \in [0, 1]$ is a step size parameter.

Hierarchical decomposition is also considered for coordination (118, 123). A bi-level optimization is considered in (124), with the upper level optimizing aggregate setpoints of DER groups and the lower level disaggregating the setpoints across individual DERs. Different from spatial decomposition, (118) proposes a temporal decomposition based on a time-varying bi-level optimization problem, which links day-ahead and real-time markets to manage DER uncertainties using distinct optimization techniques for different time scales. When linking different timescales, establishing a clear connection is crucial, e.g., using the solution from one timescale as a reference for the other (118). Uncertainties can have different characteristics and impacts depending on the timescale, and appropriate techniques (e.g., distributionally robust optimization for day-ahead planning, online optimization for real-time operation) should be employed accordingly.

5.1.2. Handling Time-Variation and Constraints.

A common technique is based on primal-dual gradient dynamics, which can be viewed as the path traced by the primal and dual variables that satisfy the KKT conditions as the system parameters vary over time (8). To make the dynamic system distributed, the key challenge is to decouple the subproblems while ensuring coordination. For instance, (116) reformulates the dynamic system using the first-order optimality conditions for each subproblem, with the boundary variables appearing as parameters. Agents communicate with a coordinator by sending quadratic approximations of their objective functions with respect to the boundary variables' increments; the coordinator solves for the optimal increments and sends them back to each agent.

Time-varying conditions can be also handled by re-solving the MPC problem at each time step for a receding horizon, which uses predictive models to deal with uncertainty and ensure feasibility (120, 123). To address multi-period constraints, methods based on Lyapunov optimization (125–127) and online convex optimization (OCO) (8, 128–130) have been developed . Prediction models for future power flow states can also be used (117).

5.1.3. Information Exchange and Local Computation.

As with most distributed optimization, a central coordinator (e.g., distribution management system (114, 119), network operator/aggregator (113, 124)) is often needed. Agents may send information pertaining to their actions (113) and boundary variables (112) to the coordinator. The coordinator may broadcast information such as dual variables (e.g., incentive signals (113, 118)) or primal variables (e.g., setpoint commands (115)), and perform real-time measurements to monitor constraint violations (118). Networked exchange between peers may include boundary information, such as power flow states and increments of DER state (121) or output powers (117), or local objective value estimates to achieve consensus (131).

Each agent's local computation often prefers simple methods such as projected gradient (131) or closed-form expressions (114, 116) derived from KKT conditions. This is often achieved by leveraging appropriate linear approximations of the AC power-flow equations (112, 114, 121, 132). A fixed number of iterations of the distributed algorithm using the previous solution as a warm start may be performed (113). In principle, ADMM with low-iteration complexity (Sec. 3.1.2) can be used (112) with transferable convergence analysis.

5.1.4. Other Practical Considerations.

Unbalanced three-phase distribution systems can be handled with an inter-phase coordination strategy (121). Non-ideal communication, such as delayed/outdated communication (119, 121, 133) and packet drops (121), has been evaluated. For instance, (133) shows that moderate delays lead to some suboptimality but not instability. Adaptive step size tuning is shown to accelerate convergence and avoid oscillations (119). The impact of network size, number of consensus steps, and gradient bias on convergence has also been assessed (131). Local iterative updates based on gradient estimates and projection are developed to optimize the steady-state performance of a networked nonlinear system while circumventing local sensitivities and satisfying input constraints (131). For real-time distributed equilibrium seeking, see also (56) and (95).

5.2. Discussion of ML for Distributed RT-OPF

CMDP: An MDP with additional costs $\{c_i\}_i$, where a feasible policy π must satisfy $V_{c_i}(\pi) \leq 0$ for all i, with $V_{c_i}(\pi) = \mathbb{E}_\pi[\sum_{t=0}^{\infty} \gamma^t c_i(s_t, a_t)]$ being the expected cumulative discounted cost under π.

ML methods (Sec. 4) are inherently suitable for RT-OPF due to the fast response capability of the learned policies based on system states (e.g., (134)). MARL techniques (Sec. 4.3) rely on localized information and are inherently suitable for distributed counterparts. For instance, (135) demonstrates the real-time computational feasibility, with an online execution time of about 40ms for a 123-bus system.

Most papers apply offline-trained RL policies (e.g., RL (103) or safe RL methods (104, 136–138)) for online control without further adaptation. If the environment is non-stationary, the offline-trained policy may not perform optimally. Extensive pretraining with diverse conditions may help handle non-stationary environments (134, 137). Extensive pretraining with diverse conditions may help (134, 137). For instance, (134) considers uncertainties from renewable energy sources and $N-1$ topology changes during training, making the trained agent more robust during online implementation.

Safety and stability are primary concerns in RT-OPF. Control-theoretic approaches can be used for stability-certified RL (76, 139). For safety constraints on state/action spaces, common approaches include penalty-based methods or using Lagrangian to derive primal-dual algorithms (134, 136, 140); Constrained Markov decision process (CMDP) has gained popularity in safe RL (104, 138). A knowledge-driven action masking technique is introduced to explicitly identify critical action dimensions based on the physical model, guiding the policy exploration in the safety direction (138). A safe RL method based on Proximal-Dual Optimization-based Proximal Policy Optimization (PDO-PPO) algorithm is proposed (104), eliminating the need for manually selecting penalty weights between rewards and safety violations. A holomorphic embedding (HE) based safety layer in the RL policy can be added to ensure the operability of the control actions (140). (137) introduces a supervisor and projector framework, where the supervisor examines the safety of the actions generated by the RL agent, and the projector modifies unsafe actions with minimal modification to ensure operational safety during online control. A hybrid method to derive the actor gradients by solving the KKT conditions of the Lagrangian using power system models is proposed to improve sample efficiency (136).

For efficient utilization of computational resources and faster solution times, existing works e.g., (133), use techniques aligned with data parallelism in distributed ML.

6. Key Challenges and Prospective Directions

Scalability and Computational Efficiency The scalability challenge in power system optimization, exacerbated by DER integration, can be addressed by distributed approaches in principle. However, it is more nuanced due to the trade-off among communication overhead, potential suboptimality, and susceptibility to faulty processes. Recent comparisons among ADMM, ALADIN, ATC, and APP show that wall-time computation does not correspond well with the number of iterations due to local computation and communication overhead (141). Practical benefits require careful consideration of communication infrastructure, data sharing protocols, and the balance between local and central computational resources, desired accuracy, and convergence time. Joint consideration of communication and convergence is essential; data exchange servers can help maintain data accuracy and timeliness, and quantized messages can reduce communication overhead. Hardware-aware computation and algorithm design for low-resource settings are important.

Handling Nonstationarity, Uncertainty, and Stochasticity Renewable energy integration, dynamic loads, and evolving topologies have introduced significant nonstationarity, uncertainty, and stochasticity in power systems. These challenges are compounded by the dynamics of distributed optimization process itself, such as time-varying communication networks, asynchronous updates, communication failures, varying agent activation mechanisms, and concurrent agent learning (Sec. 4.3.1). Online and real-time distributed optimization methods leveraging real-time data and feedback are promising (Sec. 5).

Developing optimization algorithms that can dynamically adapt their parameters, such as communication topology, synchronization frequency, or penalty/proximal factors, based on detected changes in system states or agent behaviors is a promising direction. Incorporating power system domain knowledge into machine learning models can improve their data efficiency and generalization (Sec. 4.1), as demonstrated by the winning solution to the CityLearn Challenge (106). Data-centric AI emphasizes data quality for robust ML models, which could be useful for managing the integrity of distributed data.

Drawing inspiration from "antifragility", optimization/ML methods can be designed to not only withstand uncertainty and variability but actively benefit from them. Potential connections to various areas such as meta-learning, continual learning, and multi-objective/quality-diversity optimization are explored for computational antifragility (142).

Robustness vs Resilience vs Antifragility : Robustness maintains performance under perturbations; resilience recovers from disruptions. Antifragility, a paradigm for preparing for rare events (black swans), goes beyond by leveraging volatility for growth and adaptation.

Privacy Distributed optimization requiring sensitive information sharing from/among agents may raise privacy concerns, as many existing methods, such as ALADIN, involve extensive data sharing with a central coordinator, making the system vulnerable to honest-but-curious agents and external eavesdroppers (143). Differential privacy is gaining traction due to its low computational and communication complexities. Co-designing privacy mechanisms with coordination algorithms, such as carefully choosing stepsizes, weakening factors, and noise distributions, has led to algorithms with strong privacy guarantees while preserving convergence (143). Quantized messages as a form of noisy exchange can also maintain privacy and communication efficiency.

Safety, Robustness, and Cybersecurity Distributed algorithms for power systems should be secure and resilient against failures and adversarial conditions. Anomaly detection methods can be applicable. Boundary defense mechanisms leveraging network sparsity to recover regions outside attacked areas (144) is a promising yet underexplored direction. Ensuring compatibility, compliance with data protection regulations, and maintaining stability and reliability are challenges in integrating distributed optimization/learning with existing infrastructure and regulations. ML models should be safe and data-efficient, especially under distributional shift (Sec. 4). Adapting the AI model inspector framework (145), such as stress-testing with adversarial examples and checking out-of-distribution generalization, can be relevant. Just as power system equipment requires regular maintenance, ML models may need periodic re-assessment and updates to maintain robustness as conditions change; ongoing monitoring and upkeep of ML components in distributed OPF systems over their full lifecycle is crucial.

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