# Assessing the Value of Heterogeneous Elasticities for Incentive-based Residential Demand Response

**Abstract**

This paper explores the incorporation of heterogeneous elasticity values in incentive-based demand response programs to enhance the economic efficiency of Load Serving Entities. We present three distinct models—each increasing in granularity from aggregate elasticity, through appliance-specific, to both customer and appliance-specific elasticity. Energy loss and power flow equations are also considered in the proposed model as it is an essential part of the power grid. We assess the impact of tailored demand response incentives on energy consumption patterns using a test case in Essex County, New Jersey. Our results show that while appliance-specific models and incorporating customer-specific elasticities significantly reduce operational costs, benefiting both customers and the service providers. Furthermore, the study highlights the critical role of detailed elasticity information in optimizing demand response strategies, suggesting a potential direction for future research towards leveraging advanced analytics for more effective demand management.

*Keywords:* Incentive-based demand response, Residential load, Aggregate elasticity, Appliance-specific elasticity, Customer and appliance-specific elasticity

## 1. Introduction

### *1.1. Background and Literature Review*

The increasing frequency of extreme weather events poses significant challenges to the safe operation of electric power systems worldwide. When the power system's supply capacity approaches its limit, energy demand rises, increasing the risk of system failures and operational costs, and potential financial losses [1]. In 2022, the California Independent System Operator (CAISO) reported a remarkable increase in power demand due to record-high temperatures during summer heatwaves. This increase in demand placed considerable stress on the electrical grid, significantly increasing the risk of rotating outages unless consumers reduce their energy consumption to a greater extent [2]. Consequently, the need to balance real-time energy supply and demand has led to increased utilization of demand response (DR) programs. Several studies have emphasized that the heightened uncertainty in electricity generation from renewable sources could potentially destabilize the system if additional demand-side management measures are not implemented [3, 4].

In the smart grid context, DR programs are increasingly being recognized for their capability to mitigate peak loads and lower grid operational costs [5, 6]. DR represents an effective solution to address reliability and efficiency issues in the power grid, involving changes in end-consumers' electricity consumption patterns from their normal routines during peak hours [7]. These programs offer substantial benefits, including cost reduction, energy conservation, and grid stability [8, 9]. It also provides financial benefits for Load Serving Entities (LSE), entities that purchase electricity at wholesale prices and supply it at a fixed rate. When the wholesale price of electricity exceeds the flat rate charged to customers by LSE, it becomes financially beneficial for them to motivate customers to reduce their electricity usage by providing monetary rewards.

The U.S. Department of Energy reports that the residential sector accounts for more than 38% of total electricity consumption in the United States, making it a significant source of flexibility that the system can exploit. [3]. As reported by the U.S. Federal Energy Regulatory Commission, despite 80% of the potential peak load reduction being achieved by large industrial and commercial customers [1], only a small proportion has been realized by the residential sector. Thus, residential demand response holds significant potential to reduce electricity consumption and costs, given the substantial size of the residential sector and its sparse utilization.

DR programs are broadly categorized into price-based demand response (PBDR) and incentive-based demand response (IBDR)[10]. PBDR programs charge customers varying electricity prices throughout the day, whereas IBDR programs provide specific financial incentives to customers for reducing their electricity usage during peak hours [11, 12]. Research indicates that IBDR tends to be more effective than PBDR, largely due to the direct 'bonus' benefits perceived by consumers [13]. For instance, IBDR programs have been shown to significantly reduce peak load, up to 93% in some U.S. cases [14].

Several studies have advanced our understanding of IBDR programs using innovative models focused on financial incentives and demand management. The foundational works of Ghosh et al.[15] and Aalami et al. [16] have addressed structural aspects of DR programs, focusing on optimizing operational costs and integrating interruptible/curtailable loads for effective demand management. Furthermore, Zhong et al. [17]) and Li et al. [18] introduced novel consumer engagement strategies, such as the Coupon Incentive-based Demand Response (CIDR) and economic analyses of consumer coupons, encouraging consumer participation in DR programs. Although elasticity plays a crucial role in numerous DR programs, these studies have not delved into the detailed analysis of how the heterogeneous nature of consumer sensitivities to incentives affect the outcomes of these programs. This observation points to a critical gap, the need for focused research on customer-specific elasticity within the IBDR framework.

A crucial aspect of IBDR programs is to accurately model how demand changes with changes in financial incentives, an economic concept known as elasticity [19, 20]. A higher elasticity indicates that demand is more sensitive to changes in price [21]. Elasticity is utilized for load consumption analysis and forecasting, shaping the design of DR programs, particularly for small customers. Elasticity shows the relationship between utilities' financial incentives and customer load changes [22].

To further improve the understanding of residential load profiles and consumer behaviors, Asadinejad et al. [21] investigated the customer demand response behavior and elasticity under IBDR programs, analyzing residential customers in the U.S. across various appliances and thermostat settings. Their findings indicate that elasticity significantly varies among appliances, with HVAC systems demonstrating higher elasticity due to their substantial energy consumption, emphasizing the need for appliance-specific incentives in DR programs. Similarly, Shi et al. [23] proposed an integrated model that combines technical and social-behavioral factors to enhance IBDR programs, analyzing appliance usage patterns via a large-scale survey of customers in Texas and New York. Furthermore, Lu et al. [24] explored the optimal bidding strategy of demand response aggregators by modeling customer responsiveness behaviors under different incentives. Pandey et al. [25] proposed an improved incentive-based DR model to assess their individual and combined effects on the system's economic and technical performance for distribution networks.

### *1.2. Contributions*

Despite the increasing implementation of residential DR programs in the U.S., participation levels remain below expectations. This under-performance is closely attributed to the lack of a comprehensive understanding and utilization of individual consumer behavioral patterns. Existing IBDR programs do not take full advantage of the reduction potential: their incentive policies do not incorporate (a) appliance-specific demand elasticity values, and (b) customer-specific demand elasticity values. In other words, incentive pricing is based on an aggregate demand model and is set to take on a same value across all customers and appliance types. This has been a reasonable choice so far due to privacy issues and lack of granular household electricity consumption data. However, with the growing proliferation of smart meters and privacy-preserving technologies, it is time to re-envision how to better utilize the vast amount of electricity consumption data towards designing more efficient IBDR programs.

In this study, we assess the value of incorporating heterogeneous elasticity values in the optimal operations of LSEs with incentive-based demand response. Three optimization problems with increasing levels of granularity related electricity consumption behavior are introduced; (i) first problem uses a single aggregate elasticity value, (ii) second problem uses appliance-specific elasticity values, and (iii) third problem uses customer and appliance-specific elasticity values to model demand. In each successive model, the LSE is also allowed to choose the incentive reward amounts with matching granularity. Furthermore, previous studies on DR [17, 18, 21] have overlooked the critical impact of transmission line losses within the distribution system. Given the implementation of DR programs in distribution systems, it is essential to account for energy losses, especially due to the high resistance-to-inductance ratios typical of low voltage networks. Thus, energy loss is accounted for in our third optimization problem (the first and second problems do not model separate customers, so we cannot include network loss into them). Note that estimating the specific elasticity values is out of the scope of this work but we refer the readers to [21].

Comparing the outputs of the above-mentioned three models allows us to uncover nuanced insights into customer model design in IBDR programs. We believe that these findings are invaluable for grid participants and policy-makers in creating more accurate and effective models for residential IBDR. The main contributions of this study are as follows:

- We model the optimal decision making process of an LSE by formulating it as optimization problems with varying levels of granularity in consumer elasticity. The comparison of the three different models reveals that the economic value of implementing an IBDR pricing scheme that is appliance-specific is significant. On the other hand, the economic value of further adding customer-specific granularity into the incentive pricing scheme is not as significant.

- The proposed optimization problems model self-owned generators and storage devices of the LSE and describe the dynamics of the electric storage devices. Furthermore, the third optimization problem (which has the highest granularity) models a realistic grid setup by incorporating transmission line losses through branch power flow equations. Through detailed modeling of the electric grid's dynamics and operational constraints, we are able to analyze the intricate interplay between different grid components (e.g. locational marginal price versus storage charging status), which we can then use to enhance the operation of IBDR programs.

## 2. Problem Formulation

In Figure 1, we explain the hierarchical architecture of the incentive-based demand response as applied in the current work, featuring key components including ISO, LSEs, customers, and appliances. For LSEs, the ISO determines the price, known as the Locational Marginal Price (LMP). In the proposed model, the LSE plays a key role in managing electricity demand. At each time step, it sets an incentive price, which is then broadcast to the customers. Customers receive this incentive pricing information and autonomously decide how to adjust their appliance usage. The compensation each customer receives is a function of the incentive price and their subsequent reduction in appliance-specific load.

The LMP represents the economic value of electricity across different regions, accounting for the costs associated with losses and congestion under current operational conditions. It becomes financially consequential for LSEs when the LMP exceeds the flat rate charged to customers as it necessitates LSEs to purchase electricity at prices higher than the flat rates they offer to end-users, resulting in direct economic losses. This serves as a catalyst for LSEs to implement demand reduction strategies, which not only offset their

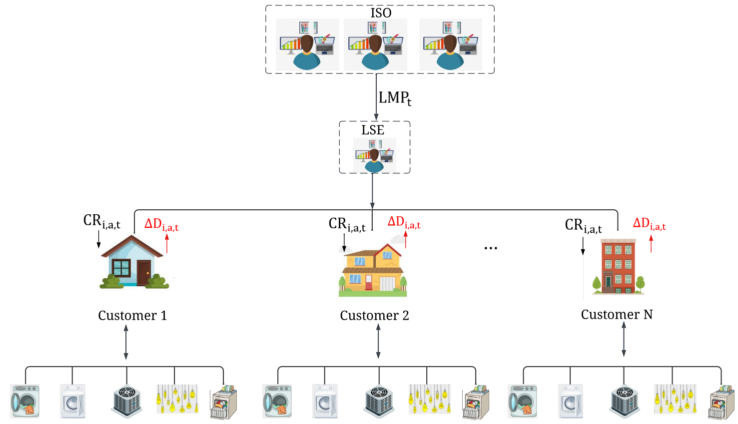


Figure 1: Hierarchical architecture of incentive-based demand response for the current work.

economic losses but also encourages efficient energy consumption, especially during peak demand periods when the system operates close to the stability margin.

In this work, we consider an LSE that operates an integrated system including self-owned generators and storage facilities. The LSE either purchases (via market by paying LMP), discharges (from storage devices), or generates electric power and injects it through the distribution feeder in order to serve its customers. The network is modeled as a distribution network (tree network structure) with AC nonlinear power flow equations and line losses. Given forecast values for LMP and estimates of the elasticity values, the LSE solves a multi-period optimization problem through which it identifies the optimal series of incentive pricing for IBDR and also the optimal operational decisions. The incentive price for IBDR is a crucial component of the LSE's objective function, aligning the management of electricity demand with economic efficiency and the behavioral patterns of the customers.

### *2.1. Model Formulation*

In our study, we present three distinct models, each with varying degrees of complexity and specificity. We use indices $i, j, k \in \mathcal{N}$ to denote network nodes, where each node acts as an aggregation of customers served by that node. In this paper, we will use the word node and customer interchangeably (i.e., node $i$ is equivalent to customer $i$). The nodes are connected via transmission lines, belonging to set $\mathcal{E}$. Each node (customer) is associated with

Table 1: Parameters

| Symbol | Description |
|---|---|
| $\boldsymbol{LMP}_t$ | Locational marginal price at time $t$ |
| $\boldsymbol{CF}$ | Fixed flat rate charged to customers |
| $\boldsymbol{D}^0_{i,a,t}$ | Base demand for customer $i$, appliance $a$, time $t$ |
| $\boldsymbol{C}_g$ | Generation cost coefficient for generator $g$ |
| $\boldsymbol{C}^c_s$ | Charging cost coefficient for storage unit $s$ |
| $\boldsymbol{C}^d_s$ | Discharging cost coefficient for storage $s$ |
| $\boldsymbol{CR}^{max/min}$ | Max/min reward value |
| $\boldsymbol{E}^{max/min}_s$ | Max/min energy level for storage $s$ |
| $\boldsymbol{X}^{max}_g$ | Max generating capacity for generator $g$ |
| $\boldsymbol{P}^{c,\max}_s$ | Max charging power level for storage $s$ |
| $\boldsymbol{P}^{d,\max}_s$ | Max discharging power level for storage $s$ |
| $\boldsymbol{\eta}^c_s$ | Charging efficiency rate for storage $s$ |
| $\boldsymbol{\eta}^d_s$ | Discharging efficiency rate for storage $s$ |
| $\boldsymbol{\epsilon}, \boldsymbol{\epsilon}_a, \boldsymbol{\epsilon}_{i,a}$ | Elasticity (varying levels of specificity) |
| $\boldsymbol{\phi}, \boldsymbol{\phi}_a, \boldsymbol{\phi}_{i,a}$ | Reduction ceiling (varying levels of specificity) |
| $\boldsymbol{r}_{i,j}$ | Resistance of line connecting node $i$ and $j$ |
| $\boldsymbol{x}_{i,j}$ | Reactance of line connecting node $i$ and $j$ |

multiple appliances, indexed by $a \in \mathcal{A}$. The LSE owns a set of generators $g \in \mathcal{G}$ and storage units $s \in \mathcal{S}$, each with its specific characteristics and constraints. Time periods are indexed by $t \in \mathcal{T}$.

Model 1 lays the foundation by utilizing an aggregate elasticity value ($\boldsymbol{\epsilon}$). This model provides a broad, collective perspective on total demand but lacks the details of individual appliance or consumer-specific behaviors.

Model 2 enhances our analysis by integrating appliance-specific elasticity ($\boldsymbol{\epsilon}_j$). It offers a more detailed view by accounting for the demand for each appliance individually. In this model, the focus shifts to understanding how the variation in elasticity values across different appliances impacts the optimal incentive pricing for IBDR. Model 3, the most detailed model, incorporates both customer and appliance-specific elasticity($\boldsymbol{\epsilon}_{i,j}$). This model enables us to formulate demand reduction with precision for each customer-appliance combination, offering critical insights for LSEs to optimize incentive pricing and reduce costs effectively. While all three models offer valuable perspectives, our study will focus on Models 2 and 3, as they provide a more detailed and granular analysis of electricity demand crucial for effective load management. They also enable us to understand the impact of modeling appliance

Table 2: Decision Variables

| Symbol | Description |
|---|---|
| $D_t$ | Demand at time $t$ |
| $D_{i,t}$ | Demand for customer $i$, time $t$ |
| $D_{i,a,t}$ | Demand for customer $i$, appliance $a$, time $t$ |
| $\Delta D_{i,a,t}$ | Demand reduction for customer $i$, appliance $a$, time $t$ |
| $CR_{i,a,t}$ | Incentive reward for customer $i$, appliance $a$, time $t$ |
| $Y_t$ | Amount of purchased electricity at time $t$ |
| $X_{g,t}$ | Generated electricity for generator $g$, time $t$ |
| $P^c_{s,t}$ | Charging power level for storage $s$, time $t$ |
| $P^d_{s,t}$ | Discharging power level for storage $s$, time $t$ |
| $P_{s,t}$ | Net power level for storage $s$, time $t$ |
| $E_{s,t}$ | Stored energy level for storage $s$, time $t$ |
| $P_{i,j,t}$ | Active power from node $i$ to node $j$ at time $t$ |
| $Q_{i,j,t}$ | Reactive power from node $i$ to node $j$ at time $t$ |
| $I_{i,j,t}$ | Complex current from node $i$ to node $j$ at time $t$ |
| $D^{img}_{i,t}$ | Reactive demand on node $i$ at time $t$ |
| $V_{i,t}$ | Complex voltage on node $i$ at time $t$ |
| $\text{loss}_t$ | Total network loss at time $t$ |

and consumer-specific elasticity, uncovering nuanced patterns and insights. The parameters and decision variables used throughout the paper can be found in Tables 1 and 2.

### *2.2. Power flow and storage constraints*

The resistance to reactance ratios in distribution systems are large compared to that of transmission systems, which lead to significant line losses. Therefore, in our study, we incorporate power flow constraints and line losses into the DR program for residential load management using the branch flow model [26, 27, 28]. Unlike traditional bus injection models that focus on nodal variables such as bus current and power injections [29], the branch flow model emphasizes the currents and power flows on individual branches [28, 30]. The branch flow model's emphasis on branch-specific dynamics allows for a more convenient modeling of power flow and loss within radial distribution networks[28].

The power flow equations are presented in equation (1) through (6). Equation (1) defines the relationship between voltage, current and apparent power. It is important to note that this equation represents a convex relaxation of the original equality constraint. As detailed in [26, 27], this relaxation is shown to be exact under certain conditions. Equations (2) and (3) represent the real power balance and the reactive power balance, respectively. The Voltage difference across the grid is expressed in equation (4). The input feeder is modeled in equation (5). Equation (6) quantifies the total losses within the distribution grid served by the LSE.

$$\frac{P_{i,j,t}^2 + Q_{i,j,t}^2}{|V_{i,t}|^2} \leq |I_{i,j,t}|^2 \tag{1}$$

$$P_{i,j,t} = \sum_{k:(j,k)\in\mathcal{E}} P_{j,k,t} + \boldsymbol{r}_{i,j}|I_{i,j,t}|^2 + D_{j,t} \tag{2}$$

$$Q_{i,j,t} = \sum_{k:(j,k)\in\mathcal{E}} Q_{j,k,t} + \boldsymbol{x}_{i,j}|I_{i,j,t}|^2 + D_{j,t}^{img} \tag{3}$$

$$\begin{aligned} |V_{i,t}|^2 - |V_{j,t}|^2 = {} & 2(\boldsymbol{r}_{i,j}P_{i,j,t} + \boldsymbol{x}_{i,j}Q_{i,j,t}) \\ & - (\boldsymbol{r}_{i,j}^2 + \boldsymbol{x}_{i,j}^2)|I_{i,j,t}|^2 \end{aligned} \tag{4}$$

$$\begin{aligned} & \sum_{g\in\mathcal{G}} X_{g,t} + \sum_{s\in\mathcal{S}} P_{s,t}^d + Y_t - \sum_{s\in\mathcal{S}} P_{s,t}^c \\ & \quad - D_{0,t} = \sum_{i:(0,i)\in\mathcal{E}} P_{0,i,t} \end{aligned} \tag{5}$$

$$loss_t = \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} \boldsymbol{r}_{i,j}|I_{i,j,t}|^2 \tag{6}$$

Incorporating loss into DR programs can significantly enhance distribution system efficiency by optimizing network performance and reducing overall energy costs [31, 32]. High losses indicate that a significant portion of the generated power is not reaching the end-users, leading to wasted energy during transmission. Failure to capture loss in IBDR models will not only underestimate the amount of power that needs to be acquired at each time period, but will also lead to sub-optimal incentive pricing schemes, thereby undermining the effectiveness of IBDR programs.

As distributed energy storage and generator devices are integral to modern power systems [33, 34], we also integrate these components into our models. In doing so, our model gains a heightened capability to optimize energy distribution and effectively manage DR, leading to more robust and adaptable strategies in residential load management. The following equations describe the constraints related to LSE-owned storage and generators.

Equation (7) embeds the time-dependent transition in stored energy level. Equation (8) models the storage charging and discharging with their respective efficiency rates. Equation(9) imposes limits on energy storage. Equations (10) and (11) imply the bounds on discharging and charging power, respectively. The generation limit is set in equation (12).

$$E_{s,t} = E_{s,t-1} + P_{s,t} \tag{7}$$

$$P_{s,t} = (\boldsymbol{\eta}_s^c P_{s,t}^c - \frac{1}{\boldsymbol{\eta}_s^d} P_{s,t}^d) \tag{8}$$

$$\boldsymbol{E}_s^{min} \leq E_{s,t} \leq \boldsymbol{E}_s^{max} \tag{9}$$

$$P_{s,t}^d \leq \boldsymbol{P}_s^{d,max} \tag{10}$$

$$P_{s,t}^c \leq \boldsymbol{P}_s^{c,max} \tag{11}$$

$$X_{g,t} \leq \boldsymbol{X}_g^{max} \tag{12}$$

*2.3. Model 1: Aggregate Elasticity*

We begin by presenting Model 1, which is the simplest but lays the foundation for the following models. The LSE aims to minimize the net cost (13) comprising terms associated with purchasing cost, fixed rate received from consumers, incentive reward payments, self-generation costs, and storage operation costs.

Equations (14)-(15) ensure that the realized electricity demand is the baseline demand minus demand reduction, which relates to the aggregate elasticity, $\boldsymbol{\epsilon}$. Equation (16) imposes the demand reduction limit by using a predetermined factor $\boldsymbol{\phi}$. In practice, this factor is determined as the point from which further reduction is unlikely to happen due to essential usages. Equation (17) provides a range of incentive values that the LSE can select from and (18) states that the total supply of electricity should be equal to the demand at all times. Finally, we have the storage/generator constraints and the non-negativity constraints on all the decision variables; the latter omitted for brevity.

$$
\begin{aligned}
\min \sum_{t=1}^{T} \Big[ & \boldsymbol{LMP}_t \cdot Y_t - \boldsymbol{CF} \cdot D_t + CR_t \cdot \Delta D_t \\
& + \sum_{g \in \mathcal{G}} \boldsymbol{C}_g X_{g,t} + \sum_{s \in \mathcal{S}} (\boldsymbol{C}_s^d P_{s,t}^d + \boldsymbol{C}_s^c P_{s,t}^c) \Big] \quad (13)
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.} \quad & D_t = D_t^0 - \Delta D_t && (14) \\
& \Delta D_t = \boldsymbol{\epsilon}(\frac{CR_t}{\boldsymbol{CF}})\boldsymbol{D_t^0} && (15) \\
& \Delta D_t \le \boldsymbol{\phi} \boldsymbol{D_t^0} && (16) \\
& \boldsymbol{CR}^{min} \le CR_t \le \boldsymbol{CR}^{max} && (17) \\
& \sum_{g \in \mathcal{G}} X_{g,t} + \sum_{s \in \mathcal{S}} P_{s,t}^d + Y_t = D_t + \sum_{s \in \mathcal{S}} P_{s,t}^c && (18) \\
& \text{Equations (7)-(12)}
\end{aligned}
$$

### *2.4. Model 2: Appliance-specific Elasticity*

Previous studies such as [] have shown that the elasticity values can vary significantly across different appliance types. Therefore, in what follows, we increase the fidelity of the IBDR model by incorporating appliance-specific elasticity values. In accordance to this change, equations (21)-(24) are appliance-specific counterparts of equations (14)-(17) in Model 1. Equation (20) simply states that the demand at any given time is the summation of all the appliance-specific demands. As before, we omit the non-negativity constraints in our presentation for brevity. Note that the LSE now has the option to choose different incentive rewards for different appliances.

$$\min \sum_{t=1}^{T} \Bigg[ \boldsymbol{LMP}_t \cdot Y_t - \boldsymbol{CF} \cdot D_t + \sum_{a \in \mathcal{A}} CR_{a,t} \Delta D_{a,t}$$

$$+ \sum_{g \in \mathcal{G}} \boldsymbol{C}_g X_{g,t} + \sum_{s \in \mathcal{S}} (\boldsymbol{C}_s^d P_{s,t}^d + \boldsymbol{C}_s^c P_{s,t}^c) \Bigg] \tag{19}$$

$$\text{s.t.} \quad D_t = \sum_{a \in \mathcal{A}} D_{a,t} \tag{20}$$

$$D_{a,t} = \boldsymbol{D^0_{a,t}} - \Delta D_{a,t} \tag{21}$$

$$\Delta D_{a,t} \leq \boldsymbol{\phi}_a \boldsymbol{D^0_{a,t}} \tag{22}$$

$$\Delta D_{a,t} = \boldsymbol{\epsilon}_a \left( \frac{CR_{a,t}}{\boldsymbol{CF}} \right) \boldsymbol{D^0_{a,t}} \tag{23}$$

$$\boldsymbol{CR}^{min} \leq CR_{a,t} \leq \boldsymbol{CR}^{max} \tag{24}$$

Equations (7)-(12), (18)

*2.5. Model 3: Customer and Appliance-specific Elasticity*

In Model 3, we increase the fidelity of the IBDR model one step further by incorporating customer and appliance-specific elasticity values. In accordance to this change, equations (27)-(30) are customer-appliance-specific counterparts of equations (14)-(17) in Model 1. Equation (26) simply states that the demand at any given time is the summation of all the customer-appliance-specific demands. For Model 3, we not only include the storage/generator constraints but also capture the line losses via the power flow equations of the branch flow model, (1)-(6). Again, non-negativity constraints are omitted for brevity. Note that the LSE now has the option to choose different incentive rewards for different customers and appliances.

$$\min \sum_{t=1}^{T} \Bigg[ \boldsymbol{LMP}_t \cdot Y_t - \boldsymbol{CF} \cdot D_t + \sum_{i\in\mathcal{N}} \sum_{a\in\mathcal{A}} CR_{i,a,t} \Delta D_{i,a,t}$$

$$+ \sum_{g\in\mathcal{G}} \boldsymbol{C}_g X_{g,t} + \sum_{s\in\mathcal{S}} (\boldsymbol{C}_s^d P_{s,t}^d + \boldsymbol{C}_s^c P_{s,t}^c) \Bigg] \tag{25}$$

$$D_t = \sum_{i\in\mathcal{N}} \sum_{a\in\mathcal{A}} D_{i,a,t} \tag{26}$$

$$D_{i,a,t} = \boldsymbol{D^0_{i,a,t}} - \Delta D_{i,a,t} \tag{27}$$

$$\Delta D_{i,a,t} \leq \boldsymbol{\phi}_{i,a} \boldsymbol{D^0_{i,a,t}} \tag{28}$$

$$\Delta D_{i,a,t} = \boldsymbol{\epsilon}_{i,a} \left( \frac{CR_{i,a,t}}{\boldsymbol{CF}} \right) \boldsymbol{D^0_{i,a,t}} \tag{29}$$

$$\boldsymbol{CR}^{min} \leq CR_{i,a,t} \leq \boldsymbol{CR}^{max} \tag{30}$$

$$\sum_{g\in\mathcal{G}} X_{g,t} + \sum_{s\in\mathcal{S}} P_{s,t}^d + Y_t \geq D_t + loss_t + \sum_{s\in\mathcal{S}} P_{s,t}^c \tag{31}$$

Equations (1)-(12)

## 3. Results

In this section, we implement the proposed models to a realistic test case and examine the impact of model fidelity (specific elasticity values, line losses) on IBDR programs. Section 3.1 describes the construct of the test case used for applying the proposed IBDR models. Section 3.2 offers comparative analyses of the three models, employing real-world residential data to illustrate their efficacy and applicability.

### *3.1. Description of Test Case*

In this test case, we consider a IBDR scenario in Essex County, New Jersey. The LMP data is collected from the PJM website [35]. The fixed rate ($\boldsymbol{CF}$) charged from the LSE to customers is set at \$120/MWh. The generation cost coefficient is set at \$90/MWh. Regarding storage parameters, the initial stored energy level is set at 4 MWh, with minimum and maximum storage levels of 0 MWh and 12 MWh, respectively. Charging and discharging efficiency rates are 0.9, and the cost coefficients for both charging and discharging are standardized at \$0.1/MWh, bench-marked based on the

Table 3: Appliance-specific Elasticity Values

| **Appliance** | **Elasticity** |
|---|---|
| Dishwasher | 0.13 |
| Washer | 0.27 |
| Dryer | 0.33 |
| Lighting | 0.42 |
| HVAC | 0.11 |

work of [33]. The $\boldsymbol{P}_s^{c,\max}$ and $\boldsymbol{P}_s^{d,\max}$ are set to 4 MW, while $\boldsymbol{X}_g^{max}$ is 1 MWh. The maximum and minimum values for the incentive reward $\boldsymbol{CR}^{max/min}$ are \$50/MWh and \$0/MWh, respectively. The values for $\boldsymbol{\phi}_a$ were chosen to be 0.8, 0.8, 0.8, 0.7 and 0.5 for dishwasher, dryer, washer, lighting, HVAC, respectively. The value for $\boldsymbol{\phi}$ is computed as the weighted average of $\boldsymbol{\phi}_a$, where the weights are determined by the relative demand of each appliance. Lastly, $\boldsymbol{\phi}_{i,a}$ values are randomly sampled from normal distributions with mean $\boldsymbol{\phi}_a$ and standard deviation 0.01.

We considered five different residential appliances: (1) Heating, Ventilation, and Air Conditioning (HVAC), (2) lighting, (3) dishwasher, (4) washer, and (5) dryer. The elasticity values are derived from the study conducted by [21] and listed in Table 3. We can observe that lighting exhibits the highest elasticity, with a value of 0.42. This indicates consumers are more responsive to incentives when it comes to energy savings from lighting. In contrast, the HVAC system displays the lowest elasticity value at 0.11, suggesting a reluctance among consumers to modify their heating or cooling usage in response to incentives. The elasticity values for other appliances, such as dishwashers, washers, and dryers, show variability, falling between these two extremes. The analysis underscores the necessity for DR programs to be finely attuned to the distinct usage patterns and preferences of consumers across different appliances. To generate customer-appliance-specific elasticity values, $\boldsymbol{\epsilon}_{i,a}$, we randomly sampled from normal distributions with mean $\boldsymbol{\epsilon}_a$ and standard deviation 0.02.

To generate a baseline demand data, we utilized the time-series end-use load profiles provided by the National Renewable Energy Laboratory (NREL) [8]. This dataset offers detailed insights into energy consumption patterns across various residential and commercial building types in the United States. Within this dataset, the data is segmented by building type (single-family homes, offices, and restaurants), and further categorized by end-use (heating,

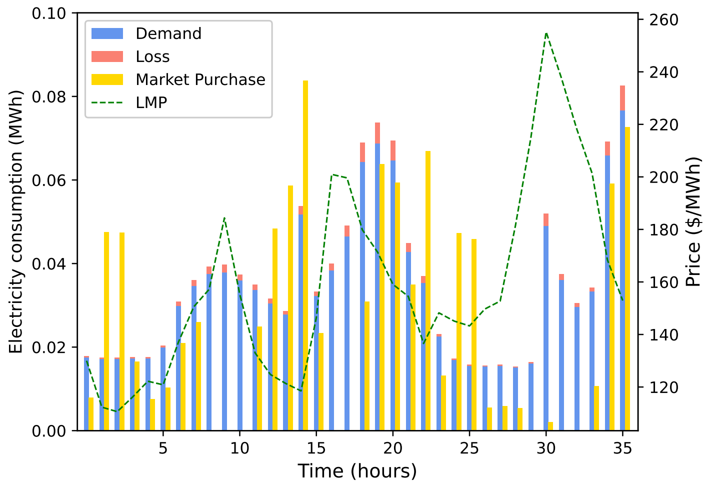


Figure 2: Analysis of LMP, demand, and market purchases against loss. Demand, loss, and market purchases are measured in MWh on the left y-axis. The LMP is represented by a green dotted line, measured in $/MWh on the right y-axis.

cooling, lighting, etc.), in 15-minute intervals. The test case used in this study was generated by overlaying demand data from 33 single-family detached buildings onto the IEEE 33-bus system (which provide the resistance and reactance values of transmission lines).

The models are evaluated over a planning horizon of one month, from January 1, 2018, to February 1, 2018, including 743 time periods. Computations were conducted on a desktop computer equipped with an Intel i7 CPU and 64.0 GB of memory, using Python 3.6.8 in the Visual Studio Code environment.

### *3.2. Comprehensive Study of the Proposed Models*

In this section, we present the findings from our case study by running three distinct models, each incorporating varying elasticity parameters to assess their impact on customer behavior.

We begin with an output from Model 3, Figure 2, which offers a comprehensive view of the overall dynamics between LMP, demand, electricity market purchases, and losses over planning time. For clarity, this figure focuses on the first 36 time periods. It is observed that loss accounts for a significant portion of the total energy, which the LSE has to account for in addition

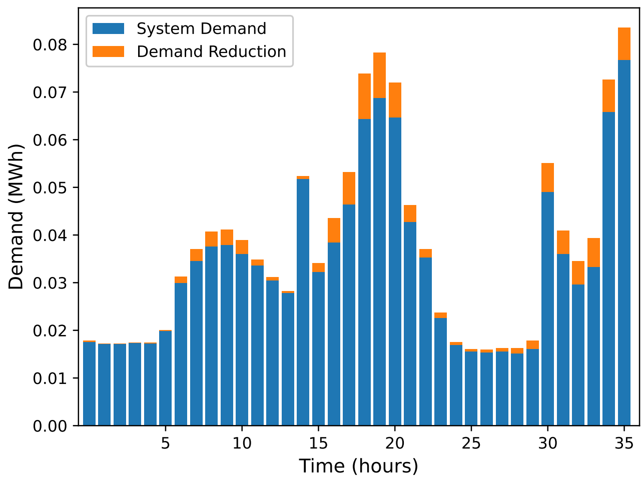


Figure 3: Analysis of total demand and its reduction.

to the actual demand. As depicted, there are instances where the amount of market purchase exceeds the actual demand plus loss. This suggests that in certain periods, additional electricity is being purchased, potentially for storage to accommodate future demand. This pattern highlights the complex decisions involved in electricity market operations, where key decisions are influenced by a variety of factors including anticipated future prices and needs, and network losses.

Figure 3 illustrates the overall demand and its reduction, with the orange bars highlighting the extent of demand reduction achieved. The summation of the blue bar and orange bar constitute the baseline demand. Figure 4 segments the total demand reduction by appliance types. It is noted that the demand reduction ratio is quantified as the demand reduction divided by the baseline demand for each appliance type. From these analyses, it becomes evident that demand reduction is most significant for lighting, attributed to customers' higher willingness to reduce its usage in response to incentives. Conversely, the dishwasher and HVAC exhibit the smallest reduction due to their lowest elasticity values.

Figure 5 presents a detailed analysis of appliance-specific reward levels. This analysis, when compared with the findings from Figure 3, reveals important insights into the allocation of incentives. Although the realized demand

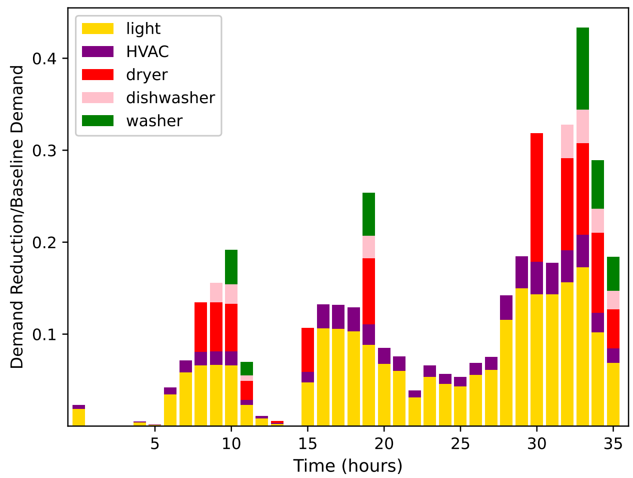


Figure 4: Realized demand reduction ratio by appliance type.

reduction ratio is most significant for lighting, as highlighted in Figure 3, Figure 5 reveals that the total incentives paid out to customers to reduce HVAC usage is comparable to that of lighting. This is attributed to the high baseline demand for HVAC and its associated low elasticity value. In other words, it is much more costly to reduce 1MWh of HVAC usage when compared to 1MWh of lighting usage. These findings highlight the cost savings potential of implementing IBDR programs with appliance-specific incentives.

Figure 6 provides a comprehensive breakdown of the various cost components within the objective function. Notably, the purchasing cost stands out as being significantly higher in comparison to the other costs. In scenarios where the selling revenue exceeds the purchasing cost plus operating costs, we observe a positive profit differential. This distinction underscores the importance of strategic purchasing and selling decisions within the market to optimize financial outcomes. On the other hand, the charging and discharging costs are found to be negligible. The incentive reward costs are notable but not overwhelming, which shows that the LSE is efficiently driving demand reduction with relatively small additional costs.

To illustrate the impact of granular elasticity values on DR program outcomes, we present the economic efficiency of the proposed models through a comparative analysis. Table 4 provides a comparison of the optimal objective

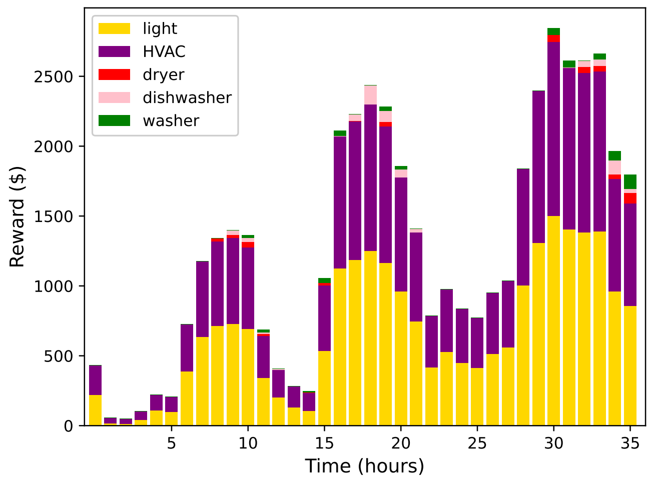


Figure 5: Total incentive reward amounts paid out to customers by appliance type.

Table 4: Cost function Values across Models

| **Models** | **Value** |
|---|---|
| Model 1 (fixed elasticity) | 1484.7 |
| Model 2 (appliance elasticity) | 1408.9 |
| Model 3 (consumer & appliance elasticity) | 1404.5 |

values achieved by each model.

From Table 4, we observe a significant reduction in the cost objective value, transitioning from Model 1 to Model 3. Specifically, transitioning from Model 1 to Model 2 yields an improvement in economic efficiency of approximately 5.11%, indicating a substantial enhancement through the implementation of appliance-specific elasticity. Further refinement in Model 3, which incorporates both customer-specific and appliance-specific elasticity, results in a modest but noteworthy improvement of approximately 0.31% over Model 2. The significant benefit from Model 1 to Model 2 highlights the initial effectiveness of appliance-specific customizations, whereas the smaller improvement from Model 2 to Model 3 indicates diminishing returns from more detailed, customer-specific elasticity.

To further elucidate the comparative cost-benefit implications for LSE across the different models studied, Figure 7 provides a detailed visual repre-

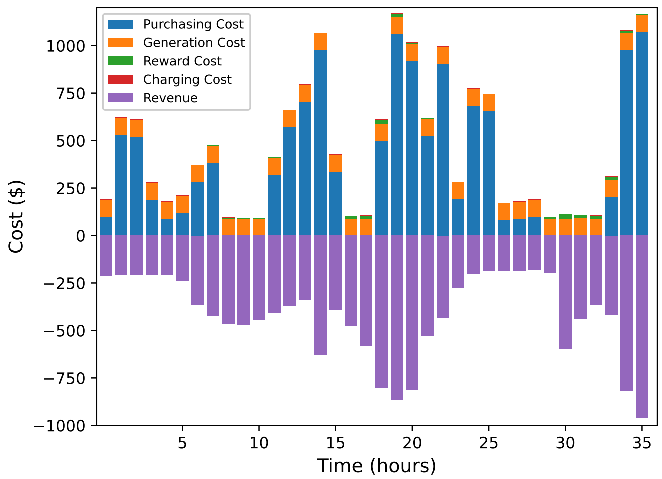


Figure 6: Breakdown of cost components within the objective.

sentation. This figure aims to clarify the economic outcomes of implementing each model, highlighting the differences in cost components such as purchasing costs, generation costs, revenue, and the total costs associated with each model.

Analysis of Figure 7 reveals that the purchasing cost is higher in Model 1 compared to Models 2 and 3, suggesting that the introduction of appliance-specific and customer-appliance-specific elasticity can lead to significant savings in energy procurement costs. Despite these variations, revenue appears relatively consistent across all models. More importantly, Models 2 and 3 are observed to incur lower total costs compared to Model 1, illustrating the economic benefits for the LSE by leveraging granular elasticity values. This approach not only enhances energy savings but also improves the financial performance of the LSE.

## 4. Conclusion

This paper presents a comparative analysis of three distinct models, each incorporating nuanced values of electricity elasticity. Model 1 employs fixed elasticity as a baseline, Model 2 introduces appliance-based elasticity to reflect the role of devices in residential life, and Model 3 extends this by including both appliance and consumer-specific elasticity, emphasizing the im-

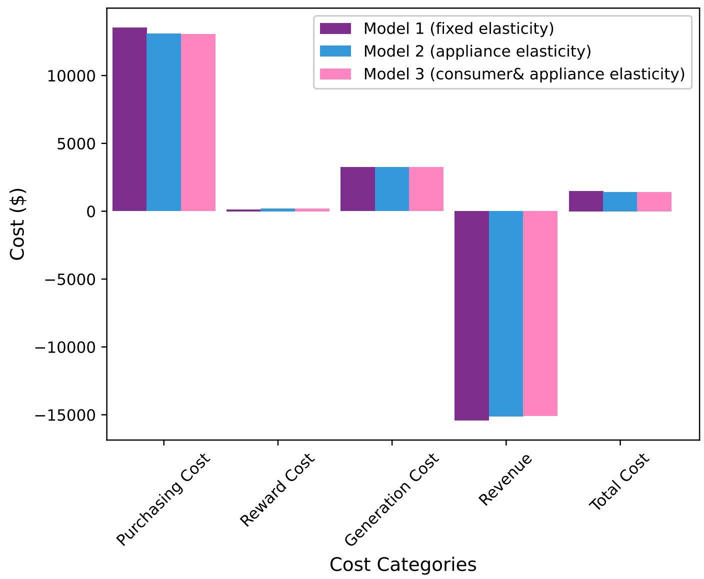


Figure 7: Analysis of Cost Components Across Models.

portance of understanding individual behaviors in DR programs. Moreover, a realistic grid setup is modeled by incorporating transmission line losses through branch power flow equations.

Our findings advocate the need and value for utilizing granular elasticity information in operating IBDR programs to achieve maximal economic efficiency by exploiting the heterogeneous responsiveness of customers regarding different appliances. Models 2 and 3 demonstrate considerable economic and operational benefits over model 1, highlighting the advantages of nuanced models for LSEs.

**Future Work**: Given the diversity in consumer consumption habits, accurately predicting users' responses is challenging, which can lead to increased total costs when user response behaviors are mischaracterized. IBDR programs often require complete consumer information; however, acquiring complete data can be difficult. Reinforcement Learning (RL) is identified as a promising approach to overcome the limitations of incomplete information[36, 37]. Future research directions include employing RL to better understand and predict consumer behaviors more effectively, aiming to optimize dynamic pricing strategies and energy consumption schedules. This direction seeks to minimize costs for service providers like LSEs while improving the efficacy of DR programs.

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